

Online Identification and Control of the Flexible Robot Arm by Recursive Prediction Error Method

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Abstract

Tending of the industry to lighter robots with higher mechanical abilities is growing day to day. The good performance of a robot needs to flexibility in robotic models which means the ability of robot in doing tasks in the wide range. This paper models and identifies an ABB industrial robot besides conducting the optimal control of it. In order to modeling, the robot arm is described by the finite number of masses that they are connected with each other by spring and dampers. The two-mass model of robot arm gives an acceptable description of the robot; at the time of rotation around an axis. The recursive prediction error method (RPEM) is improved for identifying the continuous time models. Also, LQG controller is retrofitted on the flexible robot arm for rejecting the disturbance. The obtained results from the simulation indicate the acceptable description and good performance of this controller.

Keywords:

Modeling, Online Identification, Recursive Prediction Error Method, and Optimal Control of LQR

1. Introduction

Using from industrial robots in various industries is common in manufacturing due to increasing fast, exactness, and lower cost. It can categorize the robots into two categories in a general dividing which the first category is the moving base robot (mobile) and the second category is the fixed-base robot (non-mobile). Because of ease of construction, cost, and lack of enough knowledge, the most current industrial robots with flexible robot arm is in the form of fixed base robot [1]. All calculation of position and angle related to set of the exact movement of robots is conducted by central control of the robot which sets the output signal based on input signal. The control process can be executed according to the open loop or closed loop [2].

The advantages of open loop control are lower cost and more easiness in construction and the advantage of the closed loop is its exactness. The main problem of open loop control is that if there is a disturbance in the input of system, it completely appears in the output of system. In the other hand, the problem of closed loop is the high cost of it [3]. Closed loop control can be constructed in ways like derivative, integral, proportional, and combinations of it. Considering to the factor of cost and the good return in production with quality, using the light robot with high

mechanical flexibility and having models with well performance is an inevitable problem [4].

The main issue in this paper is identification and modelling of the physical parameters in ABB IR 140 industrial robot. It is assumed that the robot arm can be approximated by the number of axes, springs, and dampers. It is clear that a model with two axes that moving around an axis can be a good description for the robot arm [1].

The identification of physical parameters of the model is conducted by on-line algorithms that they are based on the prediction of identification error [5]. Identification algorithm, a version of the recursive prediction error is used for continuous time model. The models are used for identification and control after this stage. The estimation of some physical parameters from the robot arm provides the possibility of assessment and monitoring on the important aspects of system such as load and friction. Usually, the only measurable variable in robot arm is the position of the motor shaft. In otherwise, it seems necessary having a model based on mathematical relations for compensation of flexibility in robot arm and in order to the estimation of position [6]. The optimal controller of linear quadratic gaussian (LQG) is designed after modelling and identification of the robot arm for rejecting the disturbance and assessment its performance in regulating the instability of robot arm in the presence of disturbance and keeping internal stability.

This paper has been organized as follows: in the second part, modelling of ABB flexible robot arm is conducted in way of two-mass based on Lagrangian. The third part presents the identification of ABB flexible robot arm by recursive prediction error and ARX method. Finally, the fourth part presents the designed optimal controller LQG aimed for rejecting the disturbance.

2. Modeling of ABB Flexible Robot Arm

The linear model gives the good results. So, ABB flexible robot arm can be obtained in way of Lagrangians for two-mass approximation model and later, the equations of state space are presented [7]. The considered two-mass flexible robot arm can be seen from the figure below. The mass after gear box (the middle mass) is denoted with g subtitle.

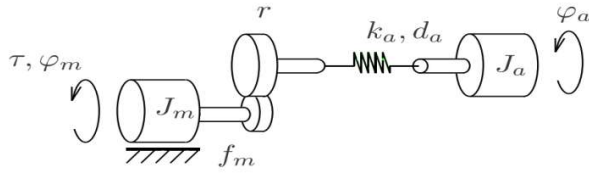


Figure 1. Two-Mass Flexible Model of Robot Arm

The used symbols in physical modelling of the robot arm are illustrated in the table below.

Table 1 Notations of Physical Modelling of Robot Arm.

Symbol	Explanation
ϕ_m	Motor Angle
ϕ_g, ϕ_a, ϕ_p	Arm Angles
f_m	Friction Coefficient of the Motor
k_g, k_a, k_p	Spring Constants
d_g, d_a, d_p	Damping Coefficient in the Spring
J_m	Moments of Inertia of the Motor
J_a, J_g, J_p	Moments of Inertia of the Arm
τ	Motor Torque
r	$\frac{1}{118}$ for One Axis of the Robot Gear Box ratio

The Euler-Lagrange equations are based on differential equations subject to holonomic constraints (constraints are equal zero). If the positions of two particles are p_1 and p_2 , the constraint can be defined as the following:

$$|p_1 - p_2| - L = 0 \quad (1)$$

Where L is the length of the bar between two particles. In contrast with the system without holonomic constraints, the number of free degrees of a system with j number holonomic constraints, the free degrees are less than j number for the late system. It can indicate the coordinate system in another independent coordinates set called generalized coordinates (ϕ_j) . These generalized coordinates are used in the following Euler-Lagrangian equation.

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{\phi}_j} - \frac{\partial L}{\partial \phi_j} = \tau_j \quad (2)$$

Where $L = K - V$ indicates Lagrangian, where K is the kinetic energy, and V is the potential energy. Generally, the obtained model from the Euler-Lagrange equation can be written as the following:

$$D(\phi) \cdot \ddot{\phi} + C(\phi, \dot{\phi}) \cdot \dot{\phi} + g(\phi) = \tau \quad (3)$$

Where $D(\phi)$ is the inertia matrix, τ is the motor torque, $g(\phi)$ represents the gravity, and $C(\phi, \dot{\phi}) \cdot \dot{\phi}$ is the coriolis and centrifugal forces. In the illustrated model in Figure 1, constant spring k_a and damping coefficient in the spring d_a is added. The equations for the two-mass flexible model are:

$$\begin{aligned} J_m \ddot{\phi}_m + f_m \dot{\phi}_m + r d_a (r \dot{\phi}_m - \dot{\phi}_a) + r k_a (r \phi_m - \phi_a) &= 0 \\ J_a \ddot{\phi}_a - d_a (r \dot{\phi}_m - \dot{\phi}_a) - k_a (r \phi_m - \phi_a) &= 0 \end{aligned} \quad (4)$$

These equations can be transformed to state space form that $y = \dot{\phi}_m$ is output signal. If $y = \dot{\phi}_m$ is the output signal, three state variables can be used instead of four state variable. The four state model is used in this description. With $y = \dot{\phi}_m$ as output signal and τ as input signal the two-mass flexible model of arm can be rewritten in state form as follows:

$$x(t) = \begin{pmatrix} r \phi_m(t) - \phi_a(t) \\ \dot{\phi}_m(t) \\ \phi_a(t) \end{pmatrix} \quad (5)$$

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (6)$$

$$A = \begin{bmatrix} 0 & r & -1 \\ -\frac{r.k_a}{J_m} & -\frac{f_m r^2 d_a}{J_m} & \frac{r d_a}{J_m} \\ \frac{k_a}{J_a} & \frac{r d_a}{J_a} & -\frac{d_a}{J_a} \end{bmatrix} \quad (7)$$

$$B = \begin{pmatrix} 0 & 0 & \frac{1}{J_m} & 0 & 0 \end{pmatrix}^T$$

$$C = (0 \quad 0 \quad 1 \quad 0 \quad 0)$$

In the linear model, $x(t)$ is as the vector of state variables, $u(t)$ denotes output signal, and $e(t)$ is white noise.

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) + K(\theta)e(t) \\ y(t) &= C(\theta)x(t) + e(t) \end{aligned} \quad (8)$$

Where $A(\theta)$, $B(\theta)$, and $C(\theta)$ are matrices that they are described by represented physical parameters in equation (8). Also, $\theta = [J_m J_a F_m K d]$ is described in equation (4). If the noise of the model is observed, K is a 5×1 matrix. In conducted assessment in the later part, the damping coefficient and noise model are considered.

3. Identification of ABB Flexible Robot Arm

The first step for control of a system is identification from the view of engineering. So, the first goal of this study is to identify of the robot arm in figure 2 by helping from input signal (i.e. reference torque φ_{ref}) and output signal (i.e. the calculated angle speed of motor in way of on-line). In this way, first, an input is driven and the corresponded output of it is immediately observed, next other inputs and outputs are driven by order.

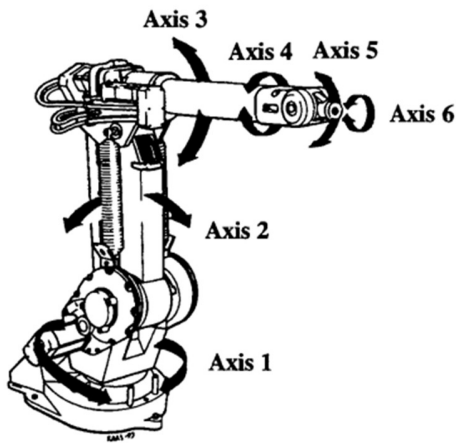


Figure 2 ABB IRB 1400 Robot.

3.1 Recursive Identification Method

It is assumed in recursive prediction error methods with N samples that the identification is before conducted.

Now, the input and output information of in $N+1$ moment are entered and the earlier estimation is modified based on the new information. In otherwise, there is a momentary relation between the system and the estimator of parameter and the parameters are updated with any new sample from input and output information. Also, it is assumed in recursive prediction error methods that the linear regression is formed and the linear regression relation is formed using N samples of input and output in the form of vector [9].

$$\begin{aligned} y_t &= u_t^T \theta + e_t \\ \begin{cases} y_1 = u_1^T \theta + e_1 \\ y_2 = u_2^T \theta + e_2 \\ \vdots \\ y_N = u_N^T \theta + e_N \end{cases} &\Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} u_1^T \\ u_2^T \\ \vdots \\ u_N^T \end{bmatrix} \theta + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} \end{aligned} \quad (9)$$

Finally, the written data in form of the linear regression in equation (9) can be rewritten in form of vector equation: $y = U\theta + e \Rightarrow \hat{\theta} = (U^T U)^{-1} U^T y$ (10)

For determining the number of samples that $\hat{\theta}$ is estimated, the estimation of input and output is denoted with $\hat{\theta}_N$:

$$y_N = U_N \theta_N + e_N \Rightarrow \hat{\theta}_N = (U_N^T U_N)^{-1} U_N^T y_N \quad (11)$$

Now, assume that sample $(N+1)$ has been entered to the computer. So, $(N+1)$ sample can be calculated as follows:

$$\begin{aligned} y_N = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} &\rightarrow y_{N+1} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \\ y_{N+1} \end{bmatrix} \& U_N = \begin{bmatrix} u_1^T \\ u_2^T \\ \vdots \\ u_N^T \end{bmatrix} \\ &\rightarrow U_{N+1} = \begin{bmatrix} u_1^T \\ u_2^T \\ \vdots \\ u_N^T \\ u_{N+1}^T \end{bmatrix} \end{aligned} \quad (12)$$

Now, earlier $\hat{\theta}$ is modified with the earlier samples and new samples:

$$\hat{\theta}_{N+1} = (U_{N+1}^T U_{N+1})^{-1} U_{N+1}^T y_{N+1} \quad (13)$$

Although this way use to new information and updated parameters, but it isn't logical that with any new sample again from the beginning the matrix U and new vector y are calculated and then new $\hat{\theta}$ is determined. While earlier $\hat{\theta}$ is obtained from the earlier samples is available, but it isn't used any. So, we should find a way that $\hat{\theta}_{N+1}$ is written as a function of $\hat{\theta}_N$ and new information i.e x_{N+1} and y_{N+1} that is called "recursive method".

$$\hat{\theta}_{N+1} = f(\hat{\theta}_N, x_{N+1}, y_{N+1}) \quad (14)$$

It can be indicated that consider to the definitions above, the structure of equation (14) is as the following:

$$\hat{\theta}_{N+1} = \hat{\theta}_N + k_{N+1}(y_{N+1} - u_{N+1}^T \hat{\theta}_N) \quad (15)$$

That it is

$$k_{N+1} = \frac{P_N \cdot u_{N+1}}{1 + u_{N+1}^T \cdot P_N \cdot u_{N+1}} \quad (16)$$

In equation above, P_N is obtained in form of recursive from the equation below:

$$P_{N+1} = P_N - \frac{P_N \cdot u_{N+1} u_{N+1}^T \cdot P_N}{1 + u_{N+1}^T \cdot P_N \cdot u_{N+1}} \quad (17)$$

3.2 Identification by Helping ARX Model

The model of system is in the form below in this method of identification:

$$A(q)y(t) = B(q)u(t - nk) + \frac{1}{1 - q^{-1}}e(t) \quad (18)$$

This method of identification is able to approximate any system provided to the degree of it is selected large [11]. The degree of ARX model is selected 7 in this study. In the following, the system of two-mass ABB IRB 1400 is identified by helping ARX model and conducting with MATLAB software and it is contrasted with black box BJ3 that is shown in figure 3.

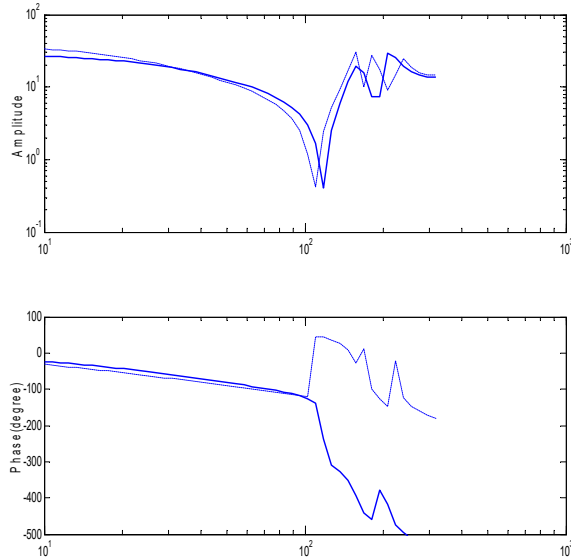


Figure 3 Bode Plots of the Identified Model (Solid: BJ3 Model and Dash Dotted: ARX Model).

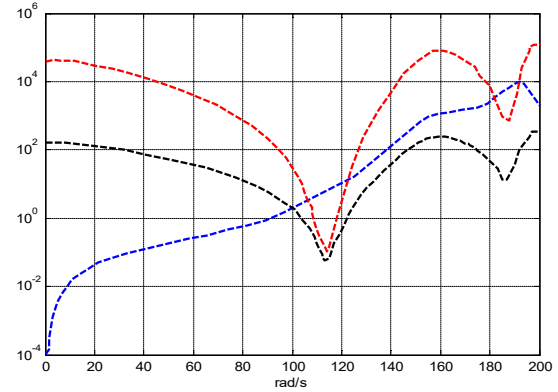


Figure 4 the Plot of $|G_\theta(i)(e^{i\omega T}, \theta)|^2$ for Three Parameters Include k_T (Black), J_a^{-1} (Blue), and f_m (Red).

3.3 Recursive Prediction Error Method

The identification algorithm in this method is RPEM with forgetting factor λ which its structure is as the following [6]:

$$\begin{aligned} \varepsilon(t) &= y(t) - \hat{y}(t, \hat{\theta}(t-1)) \\ P(t) &= \frac{1}{\lambda} \left[P(t-1) - \frac{P(t-1)\Psi^T(t)\Psi^T(t)P(t-1)}{\lambda + \Psi^T(t)P(t-1)\Psi(t)} \right] \\ K(t) &= \frac{P(t-1)\Psi(t)}{\lambda + \Psi^T(t)P(t-1)\Psi(t)} \\ \hat{\theta} &= \hat{\theta}(t-1) + K(t)\varepsilon(t) \end{aligned} \quad (19)$$

In this algorithm, matrix P is as diagonal covariance matrix and the available elements on its diagonal is deterministic in identifying of the system. Considering to importance of the parameters in this study, three parameter include f_m , J_a^{-1} , and k_T are identified which the first and second parameter are available in the structure of two-mass model where k_T is the gain static parameter of system. The obtained values for these parameters are represented besides how changing the elements of diagonal covariance matrix.

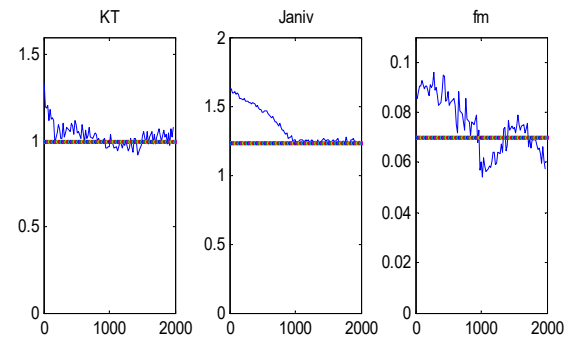


Figure 5 the Identified Parameters by Recursive RPEM Method

The solid lines that are illustrated in figure 5 indicate the reference value of parameters (optimum).

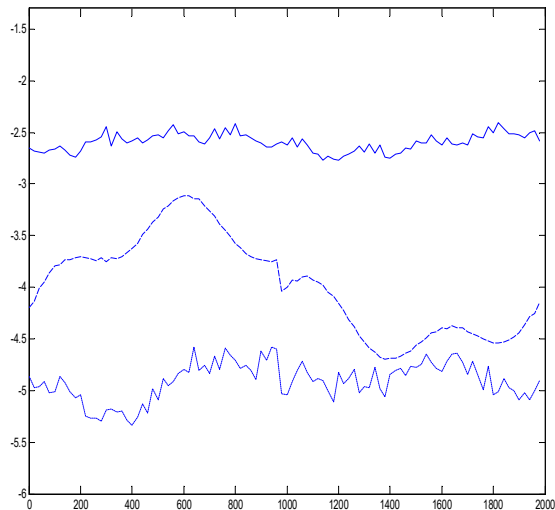


Figure 6 How Changing the Diagonal Elements of Covariance Matrix in Estimating the Parameters.

The reason of emphasizing on the covariance matrix is that it can make the identified parameters very close to optimum values by modifying it i.e. improving the elements on the main diagonal. As shown in figure 7, 3 identified parameters are illustrated after modifying the covariance matrix:

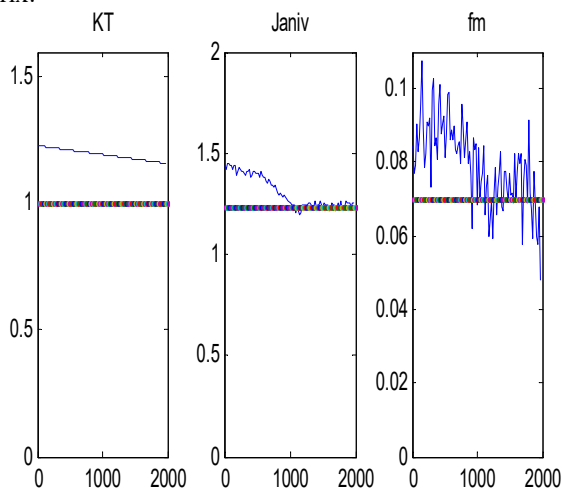


Figure 7 the Modified Identified Parameters in Recursive RPEM after Modifying the Covariance Matrix.

It can be seen from figure 8, How changing the available elements on the main diagonal of covariance matrix after modifying:

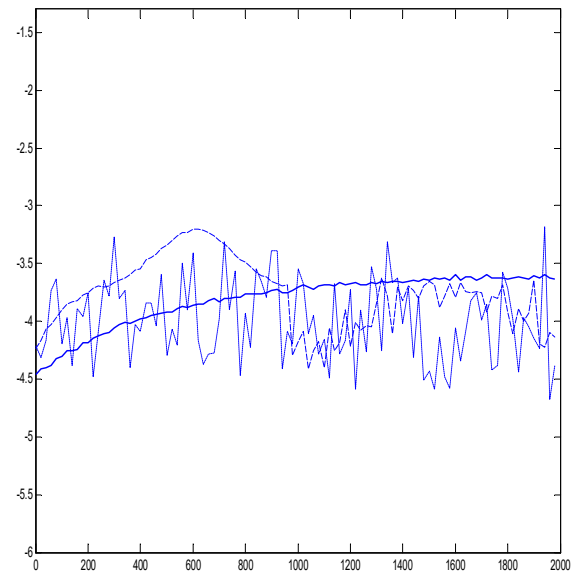


Figure 8 How Changing the Available Elements on the Main Diagonal of Covariance Matrix after Modifying

4. Designing of the Optimal Controller

Control of the second order linear regulator or LQR consists of two important parts includes LQE observer and controller LQR that the block diagram of general system in the presence of LQG controller is shown in figure 9.

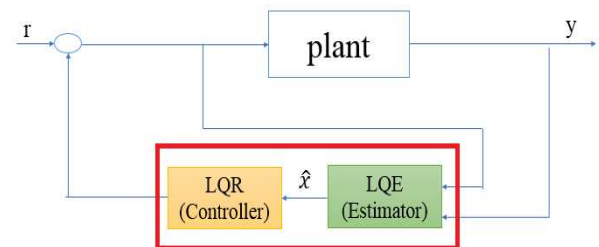


Figure 9 LQG Controller.

4.1 LQE Observer

With assuming the dynamic of a linear system as $\dot{x} = Ax + Bu$ and defining x as the state vector of system, the first step in designing the controller is the availability of the system states. It isn't possible direct access to states in many systems, so the solution is to use estimator or observer [12]. The mechanism of an observer work this way that it gets two inputs from the control signal and output signal and its output is the estimated states of system. In linear systems that the output of them is gaussian white noise, Kalman filter act optimally and present best estimation of system states. So, a part of a LQG structure is Kalman filter.

First, the linear model should be clear for estimating the states of a linear system. For doing this, consider to discrete time Gauss-Markov model(20)with the finite dimension [13].

$$\begin{cases} X_{k+1} = F_k \cdot X_k + G_k \cdot W_k \\ Z_k = H_k^T \cdot X_k + V_k (k \in \mathbb{Z}^{\geq k_0}) \end{cases} \quad (20)$$

The block diagram of the system above is shown in figure 10.

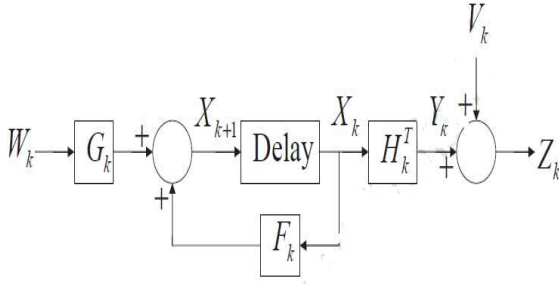


Figure 10 Block Diagram of Discrete Time Gauss-Markov Model.

Assume that the stochastic processes $\{W_k\}_{k \in \mathbb{Z}^{\geq k_0}}$ and $\{V_k\}_{k \in \mathbb{Z}^{\geq k_0}}$ and stochastic vector X_{k_0} is simultaneously normal, so we have:

$$\begin{aligned} \forall k \in \mathbb{Z}^{\geq k_0} : (E[W_k] = 0 \ \& \ E[V_k] = 0) \\ \forall k_1, k_2 \in \mathbb{Z}^{\geq k_0} : \\ (E[W_{k_1} \cdot W_{k_2}^T] = Q_{k_1} \delta_{k_1, k_2} \ \& \ E[V_{k_1} \cdot V_{k_2}^T] \\ = R_{k_1} \delta_{k_1, k_2} \ E[W_{k_1} \cdot V_{k_2}^T] = 0) \\ \forall k \in \mathbb{Z}^{\geq k_0} : (E[X_{k_0} \cdot W_k^T] = 0 \ E[X_{k_0} \cdot V_k^T] = 0) \end{aligned} \quad (21)$$

$$\begin{cases} \bar{X}_{k_0} = E[X_{k_0}] \\ P_{k_0} = cov(X_{k_0}) \end{cases}$$

In otherwise, $\{W_k\}_{k \in \mathbb{Z}^{\geq k_0}}$ and $\{V_k\}_{k \in \mathbb{Z}^{\geq k_0}}$ are the normal stochastic processes with white noise and zero mathematical expectation. First, it should be conducted first order prediction in Kalman estimator based on relation (22).

$$\hat{X}_{k+1|k} = F_k \cdot \hat{X}_{k|k} \quad (22)$$

$$\Sigma_{k+1|k} = F_k \cdot \Sigma_{k|k} \cdot F_k^T + G_k \cdot Q_k \cdot G_k^T$$

Where relation (22) uses the data in time k, then it predicts the states of system in time k+1. Also, the covariance matrix is predicted with using relation (22). After predicting with using calculation, the estimation is updated. It means that after predicting the time k+1, the calculation in time k+1 is conducted. Now, using the obtained data, the estimation in time k+1 is updated. The estimator equations of Kalman filter are illustrated in (23).

$$\begin{aligned} L_k &= \Sigma_{k|k-1} \cdot H_k \cdot (H_k^T \cdot \Sigma_{k|k-1} \cdot H_k + R_k)^{-1} \\ \hat{X}_{k|k} &= \hat{X}_{k|k-1} + L_k \cdot [Z_k - H_k^T \cdot \hat{X}_{k|k-1}] \\ \Sigma_{k|k} &= \Sigma_{k|k-1} - \Sigma_{k|k-1} \cdot H_k \cdot (H_k^T \cdot \Sigma_{k|k-1} \cdot H_k \\ &\quad + R_k)^{-1} \cdot H_k^T \cdot \Sigma_{k|k-1} \end{aligned} \quad (23)$$

Where L_k relation indicates how the calculation of the gain of estimator and the relations $\hat{X}_{k|k}$ and $\Sigma_{k|k}$ based on new data update the estimation and covariance matrix respectively.

4.2 LQR Controller

The input of this block is the estimated state of system by Kalman filter. In fact, the aim of this block is to design a control rule that it minimizes the disturbances acting on the system. The generated rule control is in the following form:

$$u = K\hat{x} \quad (24)$$

The second part of structure LQG is LQR optimal controller. Where constant K in the control rule (24) is "the optimal gain of feedback" [14]. The selected system for study is the same ABB IRB two-mass model which is in form below in the state space of system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_w w(t) + B_v v(t) \\ y(t) &= Cx(t) \end{aligned} \quad (25)$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_m} & -\frac{d+f_m}{J_m} & \frac{k}{J_m} & \frac{d}{J_m} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_a} & \frac{d}{J_a} & -\frac{k}{J_a} & -\frac{d+f_a}{J_a} \end{bmatrix} \quad (26)$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{J_m} \\ 0 \\ 0 \end{bmatrix} \quad B_w = \begin{bmatrix} 0 \\ \frac{1}{J_m} \\ 0 \\ 0 \end{bmatrix} \quad B_v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_a} \end{bmatrix}$$

Where $u(t)$, $w(t)$, and $v(t)$ are the inputs of system which they are control input, the disturbance acting on the first mass, and the disturbance acting on the second mass respectively. In fact, the aim of LQG is to reduce the impact of the disturbances on the system output [15]. Block diagram of the control system can be seen from figure 11.

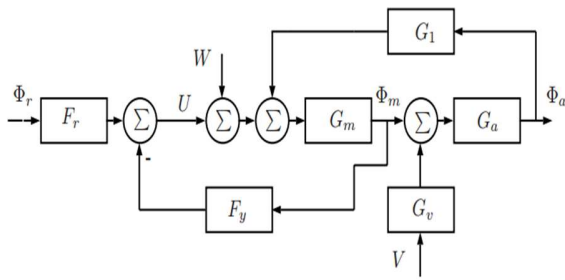


Figure 11 Two-Mass Robot Model in Presence of LQG Controller.

Transform functions G_m , G_a , G_1 , and G_v , in the control structure that is illustrated in figure 11, are the following form:

$$\begin{aligned} G_m &= \frac{1}{S^2 J_m + s(r^2 d_a + f_m) + K_a r^2} \\ G_a &= \frac{(s d_a + K_a) r}{J_a s^2 + d_a s + K_a} \\ G_1 &= r(s d_a + K_a) \end{aligned} \quad (27)$$

And the structural transform functions F_r and F_y are as follows:

$$\begin{aligned} F_r(s) &= 1 - L(sI - A + BL + KC)^{-1} B \\ F_y(s) &= L(sI - A + BL + KC)^{-1} K \end{aligned} \quad (28)$$

It requires that K and L vectors are determined for determining these functions. The main task of LQG controller is to determine these two vectors. In otherwise, K vector and L vector are determined by Kalman filter (LQE part) and LQR respectively [17]. In the following, the output of system is represented one time for $w(t)$ and one time for $v(t)$ in the presence of LQG controller.

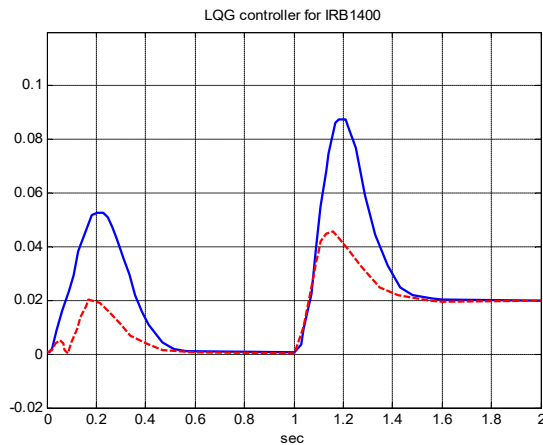


Figure 12 Output Signal Φ_a . With Inputs $w(t)$ (Blue) and $v(t)$ (Red).

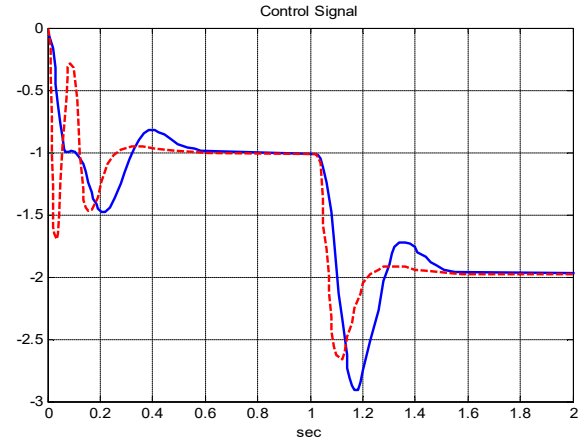


Figure 13 Control Signal. . With Inputs $w(t)$ (Blue) and $v(t)$ (Red).

The Nyquist and bode plots of transform functions G_m , G_a , G_1 , and G_v are illustrated in figure 14 to 17.

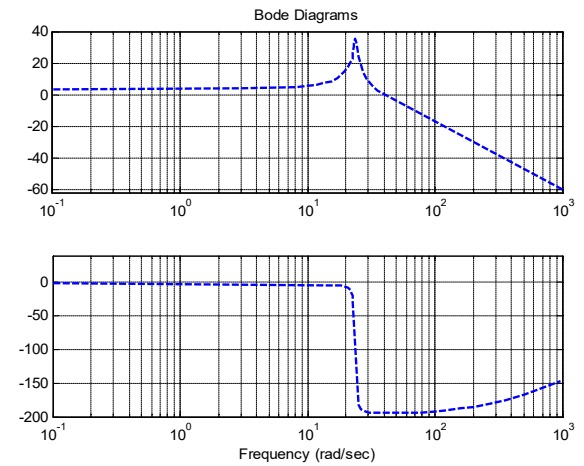


Figure 14 the Bode Plot of Transform Function $G_a(s)$.

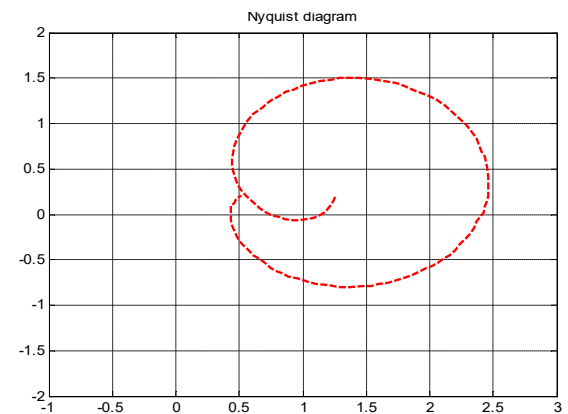


Figure 15 Nyquist Plot of $T_a(i\omega)G_a^{-1}(i\omega)$

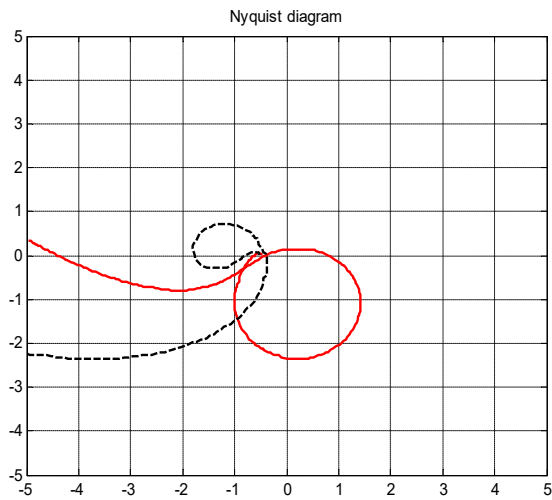


Figure 16 the Nyquist Plot of $F_y(i\omega)G(i\omega)$
(Solid: Stable Regulator and Dotted:
Unstable Regulator).

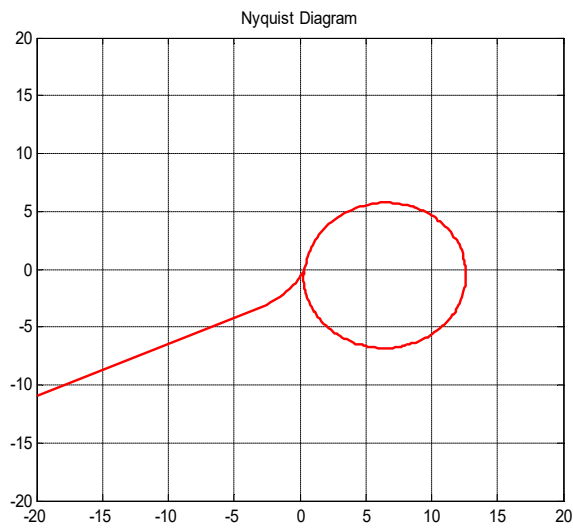


Figure 17 the Nyquist Plot of $G_0(i\omega)$.

4.3 Robustness of Designed Controller

Sensitivity function for the stable state and unstable state controller can be seen from Figure 18.

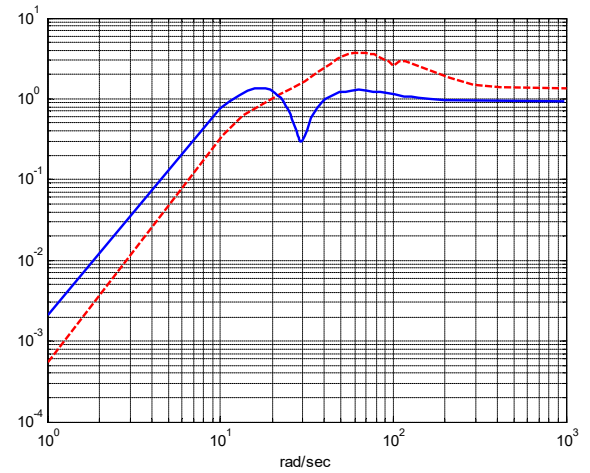


Figure 18 Sensitivity Function of Stable Regulator (Blue)
and Unstable Regulator (Red).

In the following, the obtained results from LQG robust controller are shown in Figure 19 and 20:

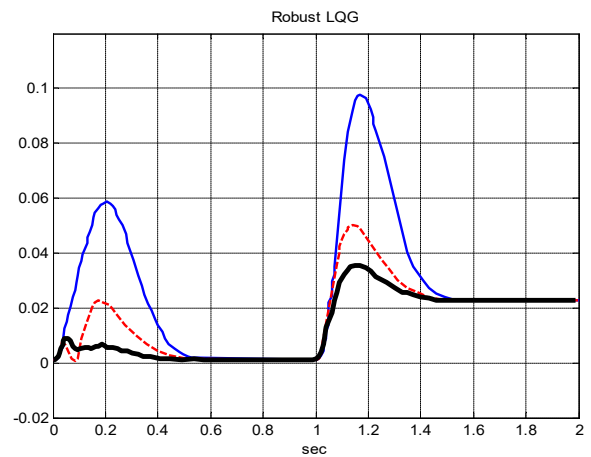


Figure 19 LQG Robust Controller (Black Solid).

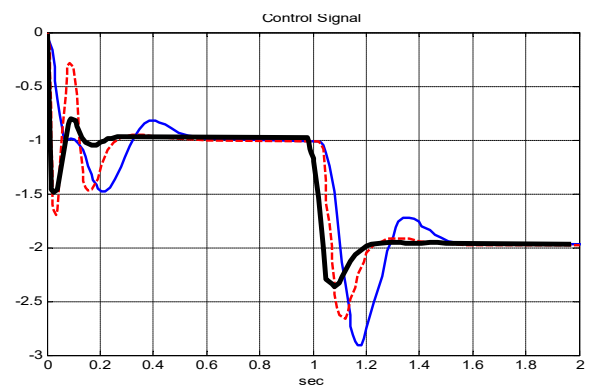


Figure 20 Robust Control Signal (Black Solid).

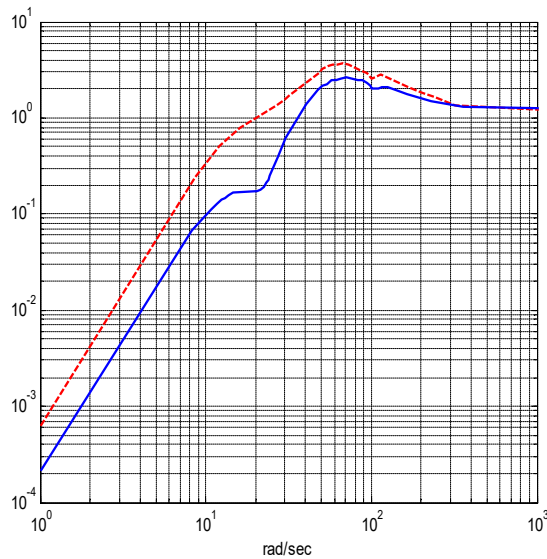


Figure 21 Sensivity Function in Presence of Robust Controller (Blue) and without Robust Controller (Red).

4.4 Designing the Controller in Presence of Noise

In the following, inputs $w(t)$ and $v(t)$, they are considered as the Gaussian white noise, and the system output in presence of LQG controller are represented:

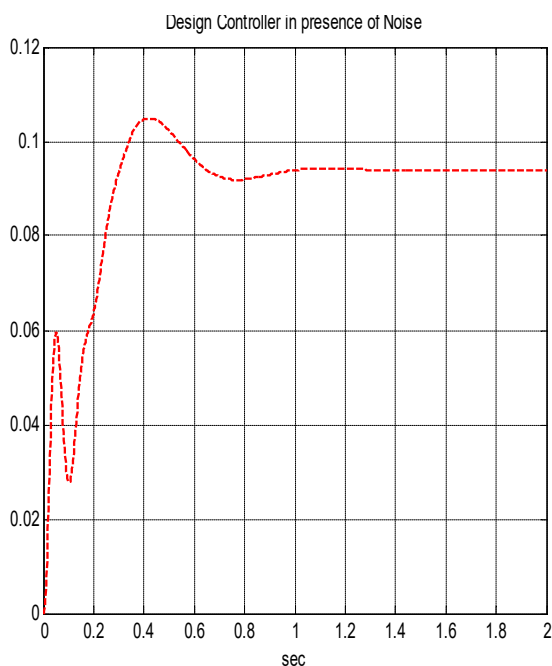


Figure 22 LQG Controller with Acting Noise Output

5. Conclusion

Modelling, identification, and control of flexible robot arm system were conducted in this study. This modeling was conducted by the finite number of mass that they connected to spring and damper. The results of simulation indicate that the two-mass model of robot arm presents an acceptable description from the robot; at the time of rotation around an axis. In order to validate the study, the obtained results from these identification methods compared with the available real data and indicates the small error that shows being suitable of the used methods. Regard to linear models in the system, the LQG optimal controller acting on the system and the results were presented. It is observed that the acted controllers ensure stability of the system besides the rejection of disturbance.

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